Characterization of information-geometric measures under correlated and uncorrelated inputs

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Background

Neurons in the brain process information by exchanging action potentials. To better understand how information is presented and processed, it is important to record many neurons from behaving animals. Multiple-electrode recordings have become an increasingly widespread tool in electrophysiology, enabling the simultaneous recording of spiking activity from tens to hundreds of neurons. These spike patterns have been analyzed by various statistical methods. Nevertheless, the detection of cell assemblies, the quantification of their correlations and estimation of synaptic interactions occurring in the underlying neural networks remain difficult problems.

Information geometry (IG) has been proposed as a novel and powerful tool for multiple neural data analysis (Amari, 2001). We have shown that the 2-neuron IG measure can infer the strength of connection weights under both correlated and uncorrelated inputs (Tatsuno et al. 2009, Nie & Tatsuno, 2012). This property is useful in neuroscience because it may provide a way to estimate the learning-induced changes in synaptic strengths from extracellular neuronal recordings. However, the influence of correlated and uncorrelated inputs to higher-order IG measures has not be investigated yet.

Objective

Our goal is to provide theoretical understanding of how the higher-order IG measures are affected by correlated and uncorrelated inputs. In addition, we also investigate how the 1-neuron IG measure is related to those inputs.

Information-geometric measure

Information-geometric (IG) measure provides a way to estimate neuronal interactions in a hierarchical manner by different orders of log linear model (LLM). For instance, the 2nd-order LLM of an N-neuron system provides the probability dist. of neuron x_1 and x_2 as,

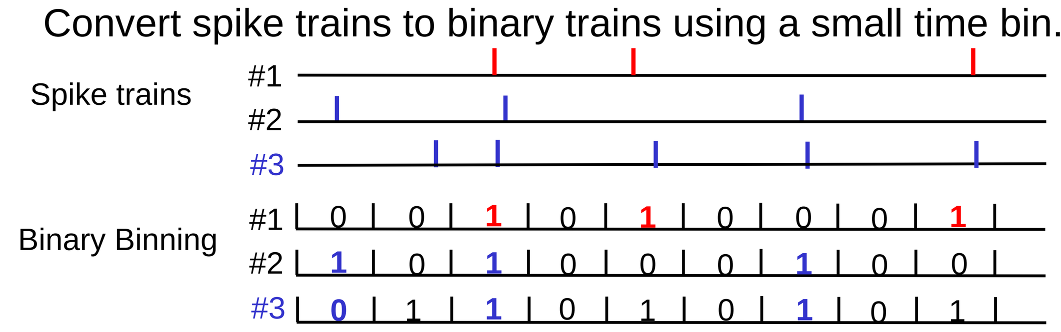
$$\log p(x_1, x_2) = \theta_1^{(k,N)} x_1 + \theta_2^{(k,N)} x_2 + \theta_{12}^{(k,N)} x_1 x_2$$

where $\theta_1^{(k,N)}, \theta_2^{(k,N)}, \theta_{12}^{(k,N)}$ represent IG measures.

$$\theta_1^{(4,N)} = \log \frac{p_{1000*}}{p_{0000*}}, \theta_2^{(4,N)} = \log \frac{p_{0100*}}{p_{0000*}}, \theta_{12}^{(4,N)} = \log \frac{p_{1100*} p_{0000*}}{p_{1000*} p_{0100*}}.$$

How to compute IG measures from spike train (2- and 3-neuron cases)

1. Convert spike trains to binary trains using a small time bin.



2. For 2-neuron system between #1 and #2, count the probability of occurrence for each pattern over all bins

$$p_{00} = \frac{n_{00}}{n}, \quad p_{01} = \frac{n_{01}}{n}, \quad p_{10} = \frac{n_{10}}{n}, \quad p_{11} = \frac{n_{11}}{n} \quad \text{where} \quad n = \sum_{i,j} n_{ij}$$

The IG measures are given by

$$\theta_1^{(2)} = \log \frac{p_{10}}{p_{00}}, \theta_2^{(2)} = \log \frac{p_{01}}{p_{00}}, \theta_{12}^{(2)} = \log \frac{p_{11} p_{00}}{p_{01} p_{10}}$$

$\theta_1^{(2)}, \theta_2^{(2)}$ are related to firing property of #1 and #2 respectively, and $\theta_{12}^{(2)}$ is a measure for connection weights.

3. For 3-neuron system, 3-neuron IG measure is calculated as,

$$\theta_{123}^{(3,3)} = \log \frac{p_{111} p_{100} p_{010} p_{001}}{p_{110} p_{101} p_{011} p_{000}}$$

which indicates a triple-wise interaction between 3 neurons.

2-neuron IG measure as a robust estimator of connection weights

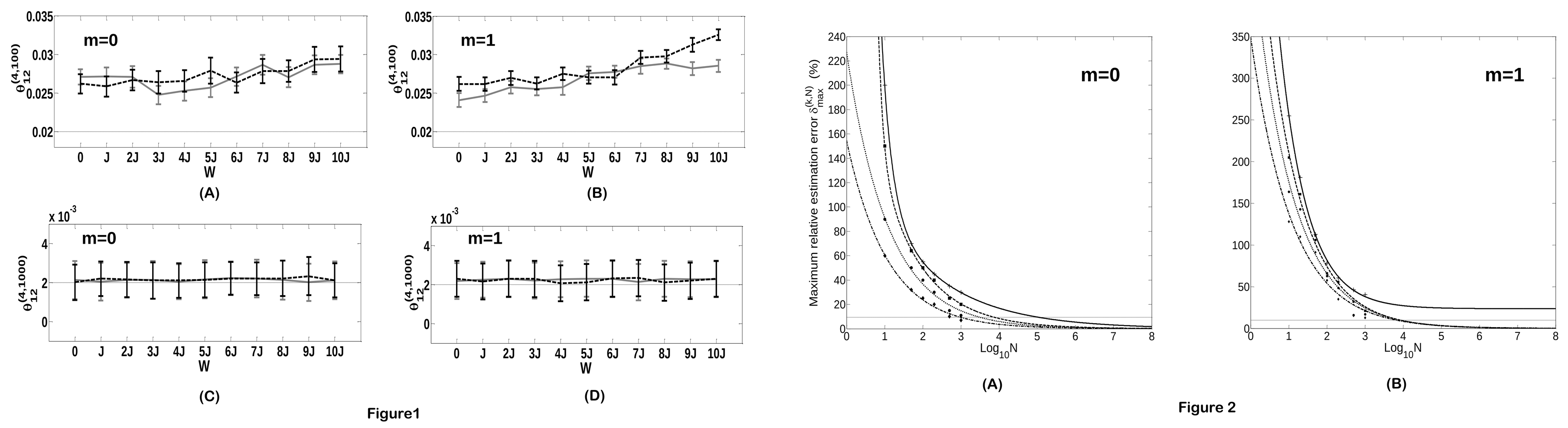
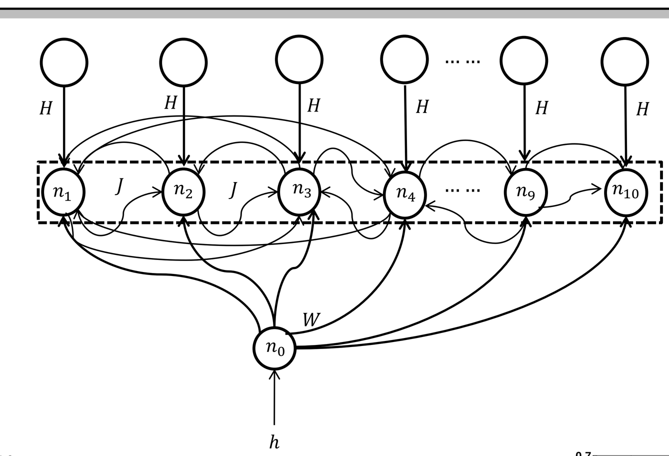


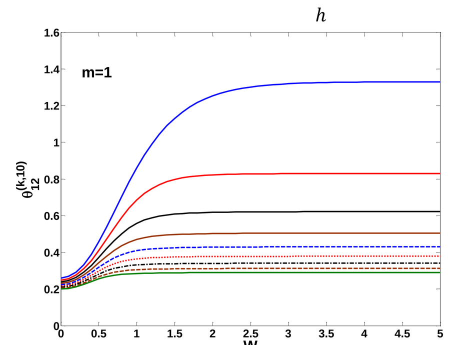
Figure1 Figure 2

Recently we investigated influence of correlated inputs on IG measures (Nie & Tatsuno, 2012). For both high (m=0) and low firing probability (m=1), the 2-neuron IG measure with 4th-order LLM, $\theta_{12}^{(4,N)}$, is affected by a correlated input for a small network (N=100, Figs 1A, 1B) but not for a large network (N=1000, Figs. 1C, 1D). Numerical simulation confirmed that $\theta_{12}^{(4,N)}$ will provide an estimation of connection strength within a 10% error for a realistically large network (N=1000 to 10000).

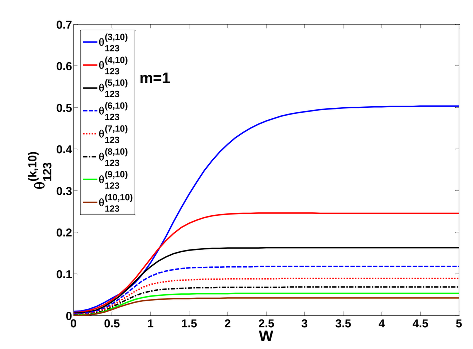
Higher-order IG measures are influenced by correlated inputs



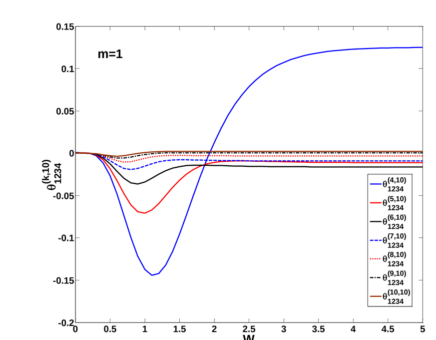
A: 10-neuron network with uniform recurrent connection (J) for theoretical calculation. The network receives both correlated input W and uncorrelated input H. The figures below use J=0.1.



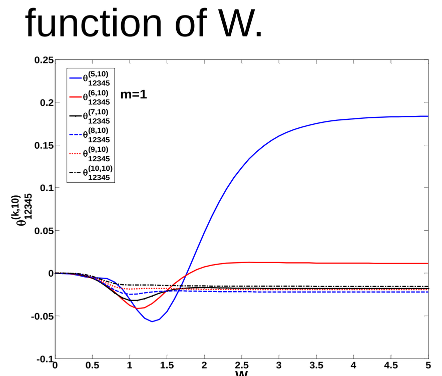
B: 2-neuron IG with LLM of different orders (2-10, from top to bottom) as the function of W.



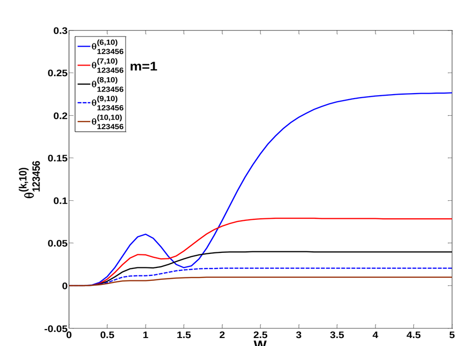
C: 3-neuron IG with LLM of different orders (3-10, from top to bottom) as the function of W.



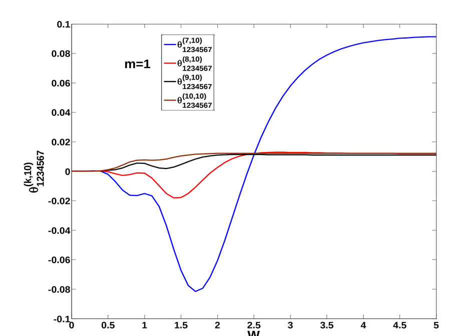
D: 4-neuron IG with LLM of different orders (4-10, from top to bottom) as the function of W.



E: 5-neuron IG with LLM of different orders (5-10, from top to bottom) as the function of W.



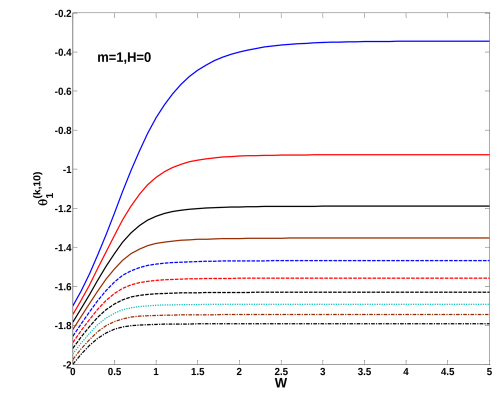
F: 6-neuron IG with LLM of different orders (6-10, from top to bottom) as the function of W.



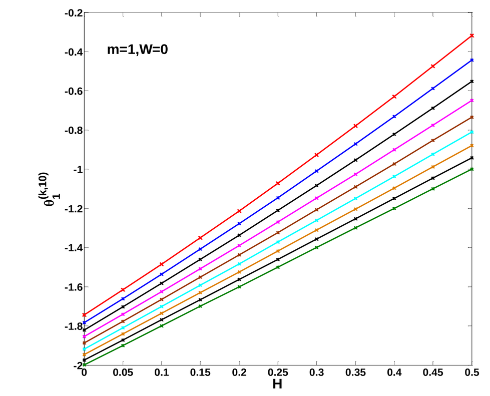
G: 7-neuron IG with LLM of different orders (7-10, from top to bottom) as the function of W.

We calculated analytical solutions for the higher-order IG measures (2-7 neuron IGs) with different orders of the log linear model (LLM). The network consists of 10 neurons with uniform recurrent connections (J), receiving both correlated inputs (W) and uncorrelated inputs (H=5J) (Figure A). Figure B shows that the 2-neuron IG measure is strongly influenced by correlated inputs (Nie & Tatsuno, 2012). Figures C-G show that the higher-order IG measures are also affected by correlated inputs in a complex way. However we found that the effect of correlated inputs becomes smaller for higher-order IG measures and higher-order LLMs.

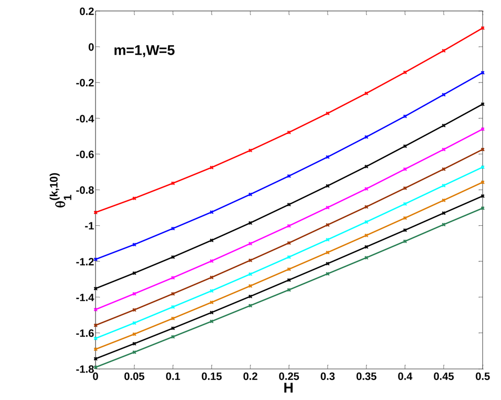
1-neuron IG measure can estimate the uncorrelated inputs



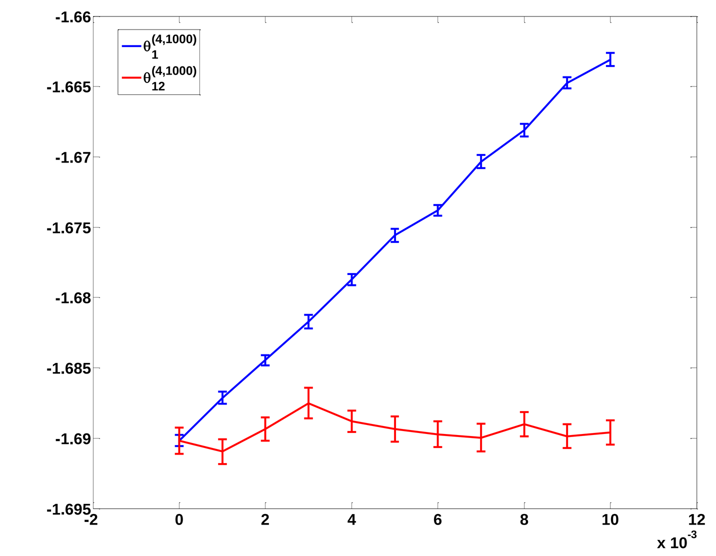
A: 1-neuron IG with LLM of different orders (1-10, from top to bottom) as the function of W



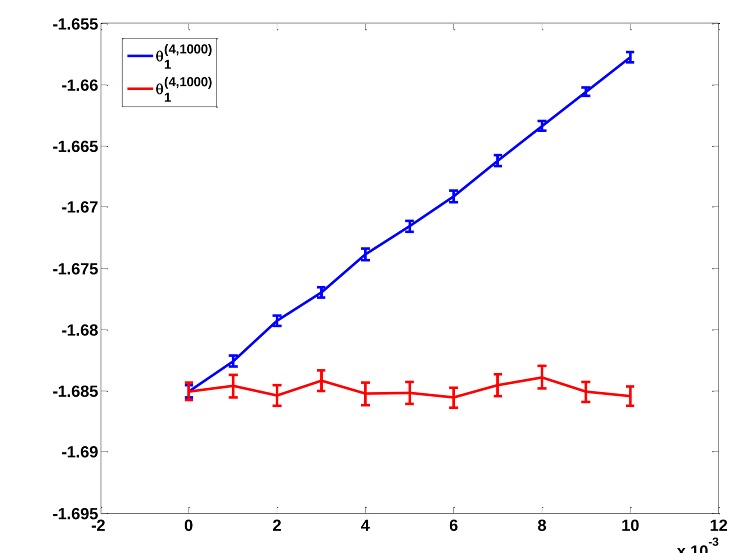
B: 1-neuron IG with LLM of different orders (1-10, from top to bottom) as the function of H (W=0).



C: 1-neuron IG with LLM of different orders (1-10, from top to bottom) as the function of H (W=50J).



D: Numerical simulation of 1-neuron IG measure with 4th-order LLM for a 1000-neuron *symmetric* network. The measure is insensitive to correlated input W (Red curve) but is linearly related to uncorrelated input H (Blue curve). The parameters are h=0.500, 2J=0.002 and m=1.



E: Numerical simulation of 1-neuron IG measure with 4th-order LLM for a 1000-neuron *asymmetric network*. The measure is insensitive to correlated input W (Red curve) but is linearly related to uncorrelated input H (Blue curve). The parameters are h=0.500, 2J=0.002 and m=1.

1-neuron IG measure is nonlinearly related to correlated inputs W in a small network (Figure A) but becomes insensitive in a large network (Figure D, red curve). 1-neuron IG measure is always linearly related to uncorrelated inputs H (Figures B, C and blue curves in Figures D, E). These results suggest that 1-neuron IG measure can estimate the relative amount of uncorrelated inputs H separately from W, even if a network receives both correlated and uncorrelated external inputs.

Summary

- Estimation of neural interaction is important for understanding information processing in the brain. However, it is influenced by correlated and uncorrelated external inputs. We investigated how the IG measures are affected by these factors.
- We found that the 2-neuron IG measure with 4th -order LLM for larger networks (N=1000-10000) is robust to correlated inputs and is able to estimate connection weights (Nie & Tatsuno, 2012).
- We found that the higher-order IG measures are also influenced by correlated inputs but lesser degrees.
- We found that 1-neuron IG measure can detect uncorrelated inputs separately from correlated inputs.
- These result suggest that the IG measures provide useful information on neural interactions.

References

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